UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

MATHEMATICS (SYLLABUS D) **4024/02**

Paper 2

October/November 2006

2 hours 30 minutes

Additional Materials: Answer Booklet/Paper
Electronic calculator
Geometrical instruments
Graph paper (2 sheets)
Mathematical tables (optional)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Section A
Answer **all** questions.

Section B
Answer **four** questions.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

Show all your working on the same page as the rest of the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 100.
You are expected to use an electronic calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

This document consists of **11** printed pages and **1** blank page.

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International Examinations

[Turn over

Section A [52 marks]

Answer **all** the questions in this section.

1 **(a)** **(i)** Factorise completely $\quad 5x^2 - 20.$ [2]

      (ii) Simplify $\quad \dfrac{5x^2 - 20}{10x^2 + 10x - 20}.$ [2]

(b) Express as a single fraction in its simplest form

$$\frac{4}{y-3} - \frac{3}{y+5}.$$ [3]

(c) Given that $\quad T = 2\pi\sqrt{\dfrac{L}{g}},$

express g in terms of π, T and L. [3]

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2 The points A and B are (5, 3) and (13, 9) respectively.

(a) Find

    (i) the midpoint of AB, [1]

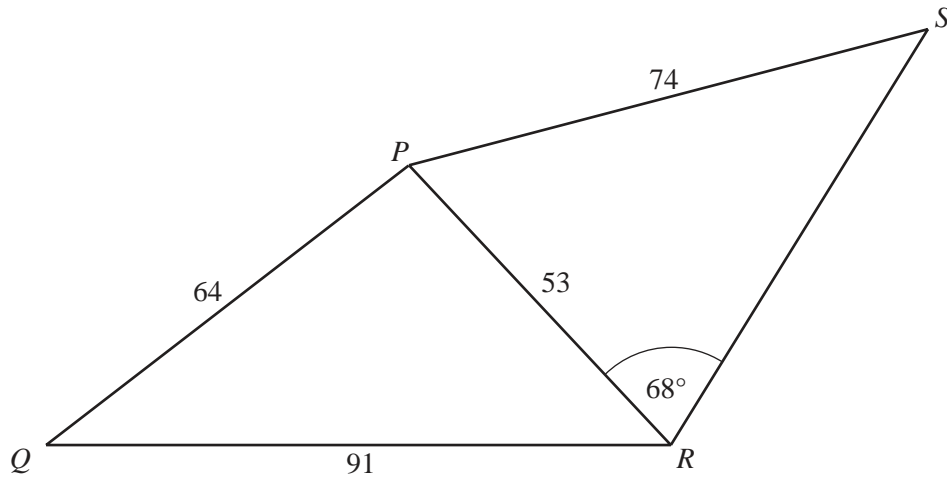   **(ii)** the gradient of the line through A and B, [1]

    (iii) the length of the line AB. [1]

(b) C is the point (–8, 5).

The point D is such that $\overrightarrow{DC} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}.$

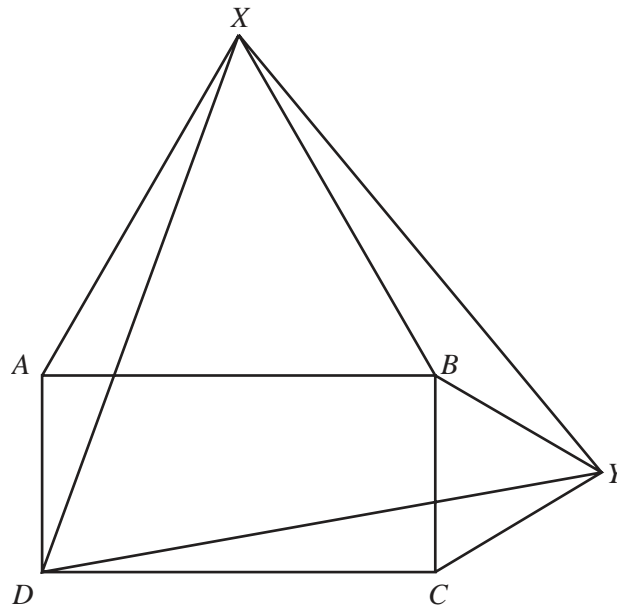   **(i)** Find the coordinates of D. [2]

    (ii) What type of quadrilateral is $ABCD$? [1]

3

The diagram shows a footpath *PR* across a park *PQRS*.
PQ = 64 m, *PR* = 53 m, *PS* = 74 m and *QR* = 91 m.
Angle *PRS* = 68°.

Calculate

(a) $Q\hat{P}R$, [3]

(b) $R\hat{P}S$, [3]

(c) the area of triangle *PRS*. [2]

4

The diagram shows a rectangle *ABCD*.

Triangles *ABX* and *BCY* are equilateral.

(a) Find $X\hat{B}Y$. [1]

(b) Show that triangles *AXD* and *BXY* are congruent. [3]

(c) Show that $D\hat{X}Y = 60°$. [2]

(d) Show that triangle *DXY* is equilateral. [2]

5 **(a)** One day the rate of exchange between pounds (£) and United States dollars ($) was £1 = $1.65.
On the same day, the rate of exchange between pounds (£) and euros was £1 = 1.44 euros.

    (i) Alan changed £500 into dollars.

    Calculate how many dollars he received. [1]

    (ii) Brenda changed 900 euros into pounds.

    Calculate how many pounds she received. [1]

    (iii) Clare changed $792 into euros.

    Calculate how many euros she received. [2]

  (b) The cost of manufacturing a television was $15 000.

    (i) It was sold to a wholesaler at a profit of 8% of the cost.

    Calculate the price the wholesaler paid for the television. [1]

    (ii) The wholesaler sold the television to a shop at a profit of 8% of the price he paid for it.

    The shop then sold the television to John at a profit of 8% of the price it paid.

    Calculate how much the television cost John. [2]

    (iii) Calculate the percentage increase in the cost of the television from its manufacture till John owns it. [2]

  (c) The shop sold a Home Entertainment system to Leslie for $46 480. The shop made a profit of 12% on the price it paid for the system.

  Calculate how much the shop paid for the system. [2]

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6 **(a)** Solve the equation $7a^2 + 12a - 11 = 0$, giving your answers correct to two decimal places. [4]

  (b) Ann drove for 4 hours at an average speed of x km/h and then for 6 hours at an average speed of y km/h.

  She drove a total distance of 816 km.

    (i) Write down an equation in terms of x and y, and show that it simplifies to

$$2x + 3y = 408.$$ [1]

    (ii) Ken drove for 3 hours at an average speed of x km/h and then for 5 hours at an average speed of y km/h.

    He drove a total distance of 654 km.

    Write down an equation, in terms of x and y, to represent this information. [1]

    (iii) Solve these two equations to find the value of x and the value of y. [3]
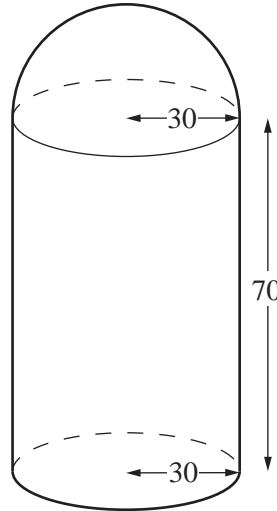

Section B [48 marks]

Answer **four** questions in this section.

Each question in this section carries 12 marks.

7   [**Surface area of a sphere = $4\pi r^2$**]
   [**Volume of a sphere $= \dfrac{4}{3}\pi r^3$**]

A hot water tank is made by joining a hemisphere of radius 30 cm to an open cylinder of radius 30 cm and height 70 cm.

(a)   Calculate the total surface area, including the base, of the outside of the tank.   [4]

(b)   The tank is full of water.

   (i)   Calculate the number of litres of water in the tank.   [3]

   (ii)   The water drains from the tank at a rate of 3 litres per second.

   Calculate the time, in minutes and seconds, to empty the tank.   [2]

   (iii)
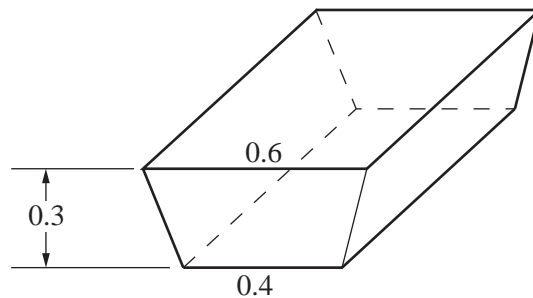
   All of the water from the tank runs into a bath, which it just completely fills.

   The bath is a prism whose cross-section is a trapezium.

   The lengths of the parallel sides of the trapezium are 0.4 m and 0.6 m.

   The depth of the bath is 0.3 m.

   Calculate the length of the bath.   [3]

8 The terms T_1, T_2, T_3, T_4, T_5 of a sequence are given as follows:

$$T_1 = \ 1 \ = 1$$
$$T_2 = \ 3 \ = 1 + 2$$
$$T_3 = \ 6 \ = 1 + 2 + 3$$
$$T_4 = 10 \ = 1 + 2 + 3 + 4$$
$$T_5 = 15 \ = 1 + 2 + 3 + 4 + 5$$

(a) **(i)** Write down the next two terms, T_6 and T_7, in the sequence

$$1, 3, 6, 10, 15, \ldots\ldots \qquad [1]$$

  (ii) The nth term in the sequence is given by $T_n = \frac{1}{2} n \, (n + 1)$.

     Show that this formula is true when $n = 7$.   [1]

  (iii) Use the formula to find T_{100}.   [1]

  (iv) Use your answer to part **(iii)** to find $5 + 10 + 15 + \ldots\ldots + 500$.   [1]

  (v) Hence find the sum of all the whole numbers from 1 to 500 which are **not** multiples of 5. [2]

(b) The terms S_1, S_2, S_3, S_4, S_5 of a different sequence are given as follows:

$$S_1 = \ 1 \ = 1 \times 1$$
$$S_2 = \ 4 \ = 1 \times 2 \ + \ 2 \times 1$$
$$S_3 = 10 \ = 1 \times 3 \ + \ 2 \times 2 \ + \ 3 \times 1$$
$$S_4 = 20 \ = 1 \times 4 \ + \ 2 \times 3 \ + \ 3 \times 2 \ + \ 4 \times 1$$
$$S_5 = 35 \ = 1 \times 5 \ + \ 2 \times 4 \ + \ 3 \times 3 \ + \ 4 \times 2 \ + \ 5 \times 1$$

  (i) Find S_6 and S_7.   [2]

  (ii) The nth term in this sequence is given by $S_n = \frac{1}{6} n \, (n + 1) \, (n + 2)$.

     Show that this formula is true when $n = 7$.   [1]

  (iii) Find $1 \times 20 + 2 \times 19 + 3 \times 18 + \ldots\ldots + 20 \times 1$.   [1]

(c) $S_2 - S_1 = (1 \times 2 \ + \ 2 \times 1) - 1 \times 1 = 1 \times 2 \ + \ 1 \times 1 = 2 \ + \ 1 = T_2$
        $S_3 - S_2 = (1 \times 3 \ + \ 2 \times 2 \ + \ 3 \times 1) - (1 \times 2 \ + \ 2 \times 1) = 3 \ + \ 2 \ + \ 1 = T_3$
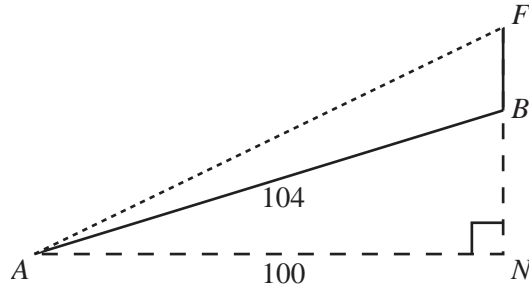
Show that

  (i) $S_4 - S_3 = T_4$,   [1]

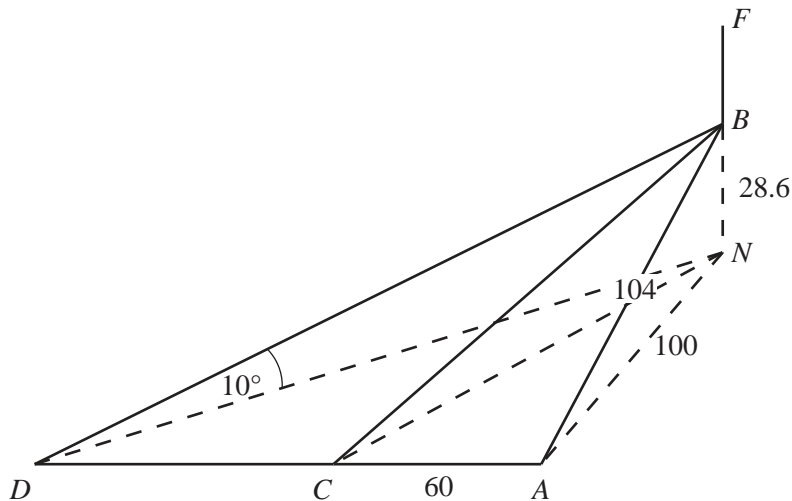 **(ii)** $S_{n+1} - S_n = T_{n+1}$.   [1]

9 A vertical flagpole, *BF*, stands at the top of a hill.
   AB is the steepest path up the hill.
   N lies vertically below *B* and $A\hat{N}B = 90°$.

   AN = 100 m and *AB* = 104 m.

(a) Show that *BN* = 28.6 m. [1]

(b) It is given that $F\hat{A}N = 25°$.

   (i) Write down the size of the angle of depression of *A* from *F*. [1]

   (ii) Calculate the height, *BF*, of the flagpole. [3]

(c)

   The diagram shows three other straight paths (*CB*, *DB* and *ACD*) on the hill.

   The path *ACD* is horizontal and $B\hat{A}C = N\hat{A}C = 90°$.

   CN and *DN* are horizontal lines.

   (i) Given that *AC* = 60 m, calculate $B\hat{C}N$. [4]

   (ii) Given that $B\hat{D}N = 10°$, calculate $D\hat{B}A$. [3]

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10 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation

$$y = \frac{x^2}{5} + \frac{5}{x}.$$

The table below shows some values of x and the corresponding values of y correct to 1 decimal place.

x	1	1.5	2	3	4	5	6
y	5.2	3.8	3.3	3.5	4.5	6.0	p

(a) Calculate the value of p. [1]

(b) Using a scale of 2 cm to represent 1 unit on each axis, draw a horizontal x-axis for $0 \leqslant x \leqslant 6$ and a vertical y-axis for $0 \leqslant y \leqslant 9$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to find the values of x in the range $1 \leqslant x \leqslant 6$ for which $\dfrac{x^2}{5} + \dfrac{5}{x} - 4 = 0$. [2]

(d) By drawing a tangent, find the gradient of the curve at the point (4, 4.5). [2]

(e) (i) On the same axes, draw the graph of $y = \frac{1}{2}x + 3$. [2]

(ii) Write down the x coordinates of the points at which the two graphs intersect. [1]

(iii) Find the equation, in the form $2x^3 + ax^2 + bx + c = 0$, which is satisfied by the values of x found in part **(e)(ii)**. [1]

11 **Answer the whole of this question on a sheet of graph paper.**

The diagram shows the histogram which represents the heights of the pupils in a small school.
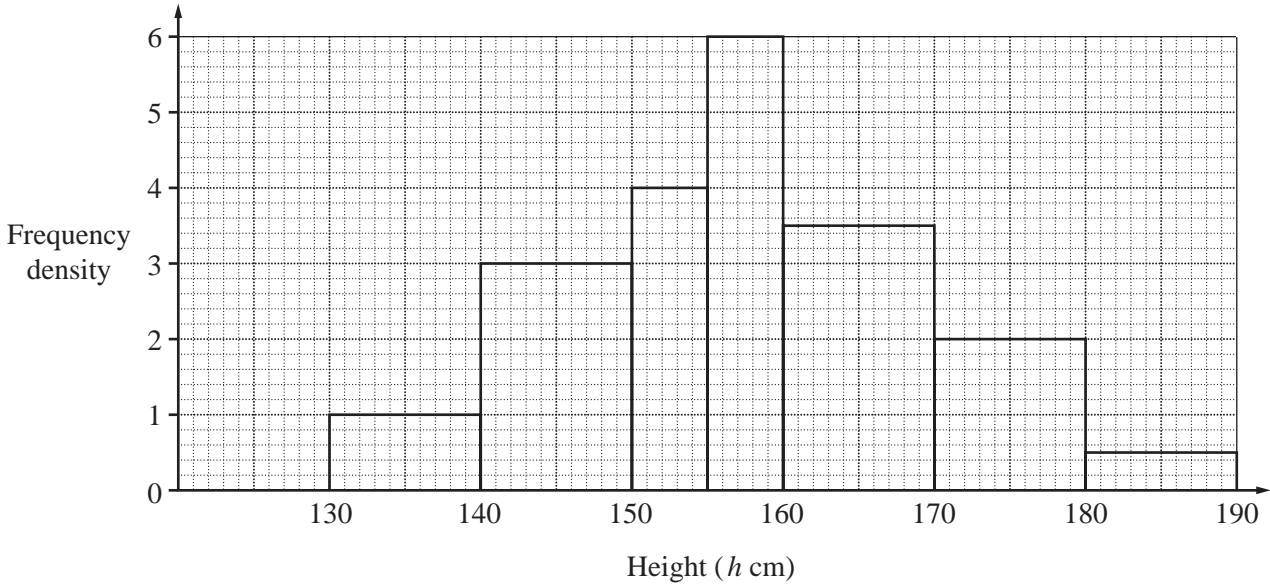
(a) **(i)** **On your graph paper**, copy and complete this frequency table that represents the distribution.

$130 < h \leqslant 140$	$140 < h \leqslant 150$	$150 < h \leqslant 155$	$155 < h \leqslant 160$	$160 < h \leqslant 170$	$170 < h \leqslant 180$	$180 < h \leqslant 190$
10						

[2]

(ii) Hence copy and complete this cumulative frequency table that represents the distribution.

Height (h cm)	$\leqslant 130$	$\leqslant 140$	$\leqslant 150$	$\leqslant 155$	$\leqslant 160$	$\leqslant 170$	$\leqslant 180$	$\leqslant 190$
Cumulative frequency	0	10						

[1]

(b) Using a scale of 2 cm to represent 10 cm, draw a horizontal h-axis for $130 \leqslant h \leqslant 190$. Using a scale of 1 cm to represent 10 pupils, draw a vertical axis.

On your axes, draw a smooth cumulative frequency curve to illustrate the information. [3]

(c) Use your graph to find

    (i) the median height of the pupils, [1]

    (ii) the lower quartile height, [1]

    (iii) the interquartile range. [1]

(d) One student is chosen at random.

Use the frequency table to find the probability that the student's height is greater than 160 cm.
Give your answer as a fraction in its lowest terms. [1]

(e) Two students are chosen at random.

Calculate the probability that one has a height greater than 170 cm and the other has a height at most 140 cm.
Give your answer as a fraction. [2]

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